Formulas of Microstrip with a Truncated Substrate by Synthetic Asymptotes—A Novel Analysis Technique

Y. Leonard Chow, *Member, IEEE,* and Wan C. Tang

***Abstract*—A substrate is usually truncated far enough to avoid disturbing the microstrip-line characteristics. Such uneconomical practice is not necessary if the disturbances as a function of truncation are known in formulas and, therefore, are easily compensated. This paper derives the desired formulas from the novel technique of synthetic asymptotes. Truncation at both sides of a line is derived here with 3% average error. For the practical one-sided truncation at the edge of a chip or circuit board, the average error is only half at 1.5%.**

***Index Terms*—Microstrip, synthetic asymptote, truncated substrate.**

I. Introduction

IT IS economical to make the maximum use of a circuit board, or a chip, by printing the microstrip lines very close to the truncated edges of the substrate. This is not usually done since such closeness to edge disturbs it in the form of an increase in characteristic impedance and reduction in effective dielectric constant from the designed values.

Restoration to the designed values is possible, in principle, through synthesis by widening and lengthening the microstrip. This is tedious, however, since the disturbances are normally obtainable by numerical computation [1], [2] or lengthy space harmonics [3]. This difficulty may be overcome by providing simple formulas. Curve-fitted formulas (like those in [3]) are difficult, as there are at least three design parameters to curve fit, namely, the dielectric constant ε_r, linewidth W, and substrate truncation distance w_t; the substrate thickness h may be used as a normalization factor to distances.

This difficulty is overcome if the formulas are derived from theoretical considerations. Since they are derived, the formulas are accurate over the full range of all three parameters, and give good physical insight.

A. Principle of Synthetic Asymptote

The theoretical methods involve three methods: the synthetic asymptote [4] with the help of the classical images and multipole expansion.

The first method, i.e., synthetic asymptote, is a novel technique for microwave. Its use was first tested by one of the authors in the grounding systems in power engineering [4]–[7]. The principle of the technique is as follows. Assume that the two *regular* asymptotes of a function are analytically known, say, at a parameter's near limit of zero and far limit of infinity. The technique is then *to construct a synthetic asymptote so that it converges into the original regular asymptotes at the two limits*. This synthesized asymptote leads to the desired formula.

For asymptotes of two independent parameters (e.g., the truncating distance and substrate thickness in this paper), the synthetic asymptote derivations are to be applied twice. If the regular asymptotes themselves are not known, they are also derived.

Despite different variations [4]–[7], the principle is the same and simple—a synthetic asymptote is a formula constructed to match the original asymptotes at their limits. The average error is frequently 3% or less.

The second method is the classical charge images [8] on a regular (nontruncated) substrate. It is used in this paper to derive the regular asymptotes needed, but unavailable in the literature.

The third is the multipole expansion [9], which is useful for the case where only the asymptote value at the parameter limit is known. This regular asymptote must be a series of the multipole terms from the Laplace's equation. Only dominant terms are needed. The coefficients of the terms are then found from the one asymptotic value.

Each step in constructing a synthetic asymptote or a regular asymptote can be simple. However, there are a number of asymptotes and, hence, a number of steps. *Therefore, to avoid confusion, a flowchart of the steps is included, as shown in Fig. 1*. Each step is identified with a related section (e.g., Section II) or with the Appendix.

The flowchart and figure depicted in Fig. 1 assume that the truncation is cut symmetrically on both sides of the microstrip line. Equation (20) easily converts the formulas to allow the truncation to be on one side only. *Despite the length of the derivations, the resulted formulas are quite simple and convenient for calculation with a calculator. The accuracy of the formulas is good*.

B. Geometry

Following Smith and Chang [1], Fig. 1 assumes that the ground plane below the truncated substrate is nontruncated. This is to ensure, mathematically, a zero absolute potential on the ground plane. In practice, while a microstrip line is printed near an edge of a printed circuit board, the ground plane of the printed circuit board is still large enough to ensure its near-zero absolute potential. Further, when in operation, this ground plane is clamped to an outside ground.

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The authors are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong (e-mail: eeylchow@cityu.edu.hk).

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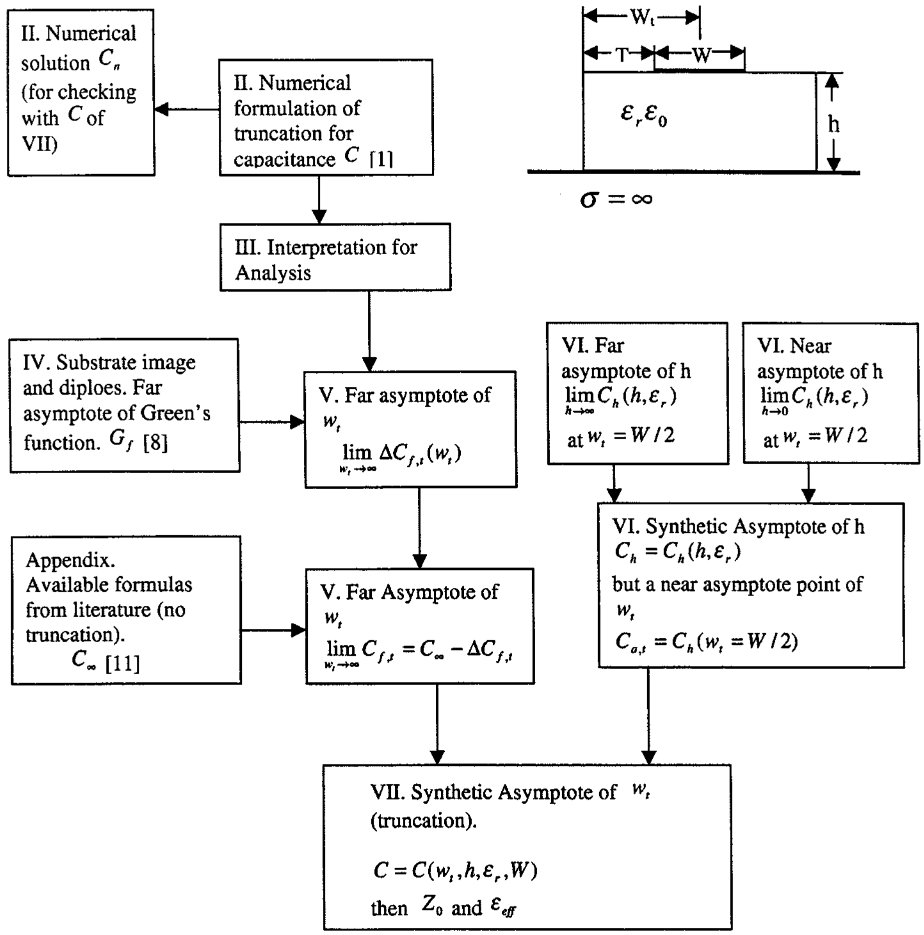

Fig. 1. Microstrip line with a truncated substrate and the flowchart for the construction of asymptotes and synthetic asymptote. Material from outside has reference numbers marked in the flowchart.

It is possible to derive similar formulas for the case of a ground plane truncated along with the substrate, like that in the subsequent paper of Smith and Chang [2]. This is not done here because the nonzero potential at a smaller truncated ground plane is less realistic.

Unlike [3], the microstrip of Fig. 1 is in an open space and not in a metal enclosure. Since the enclosure is usually large enough when compared with a chip, the enclosure causes little effect.

If the metal enclosure is small with the chip, or circuit board, butting against it, an engineer may not design a line close to the edge of the chip due to the likely low characteristic impedance.

Quasi-TEM propagation along the line is assumed for the microstrip line. The metal thickness of the microstrip line is also assumed to be negligible.

II. Formulation of Free and Equivalent Charges on Truncated Substrate

This formulation taken from [1] is an integral equation. An integral equation is normally solved numerically. In this paper, however, we shall interpret the formulation to generate the regular and synthetic asymptotes.

We now summarize the formulation. A microstrip line with a truncated substrate is shown in Fig. 1. The microstrip line with a truncated substrate in Fig. 1 has a *free* charge density $\rho_f(\mathbf{r})$ across the strip and the strip has a constant potential of 1 V.

Following Smith and Chang [1], the equivalent (free plus bound) charge density $\sigma(\mathbf{r})$ can be solved by

$$\Phi(\mathbf{r}) = \int_S \sigma(\mathbf{r}') G_p(\mathbf{r}, \mathbf{r}')\, dS' \tag{1}$$

where (1) gives a method of moments (MoM) solution and S is the interface contour boundary with the appropriate boundary condition on the dielectric and conductor surfaces. The *potential* Green's function G_p in (1) is two-dimensional (2-D) for a line and is in free space.

With the equivalent charge $\sigma(\mathbf{r})$ solved, the free charge density $\rho_f(\mathbf{r})$ is given by

$$\rho_f(\mathbf{r}) = (\varepsilon_r + 1)\frac{\sigma(\mathbf{r})}{2} + \varepsilon_0(\varepsilon_r - 1)\int_S \sigma(\mathbf{r}')\frac{\partial G_\rho(\mathbf{r}, \mathbf{r}')}{\partial n}\, dS'. \tag{2}$$

The capacitance C_n of the microstrip line with truncation is given by integrating the free charge $\rho_f(\mathbf{r})$

$$C_n = \int_S \rho_f(\mathbf{r})\, dS'. \tag{3}$$

Equations (1)–(3) complete the formulation of Smith and Chang [1].

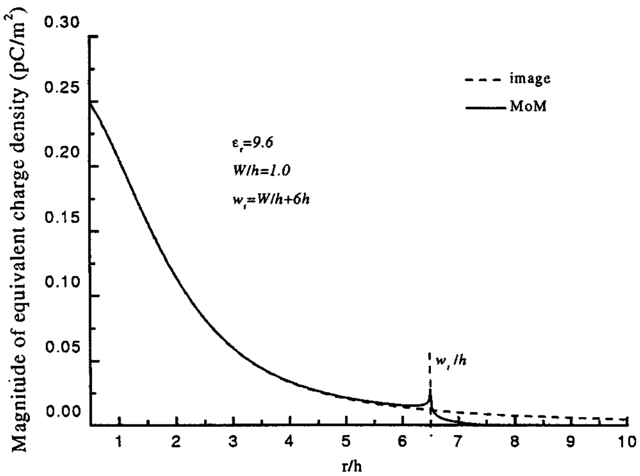

Fig. 2. Equivalent charge distribution (truncated: solid line) along the *horizontal* substrate surface ($r/h \leq 6.5$) and along the *vertical* truncated surface ($6.5 \leq r/h \leq 7.5$). It is computed with the MoM from (1). The distribution (nontruncated: dotted line) is also computed from the classical image theory with the MoM.

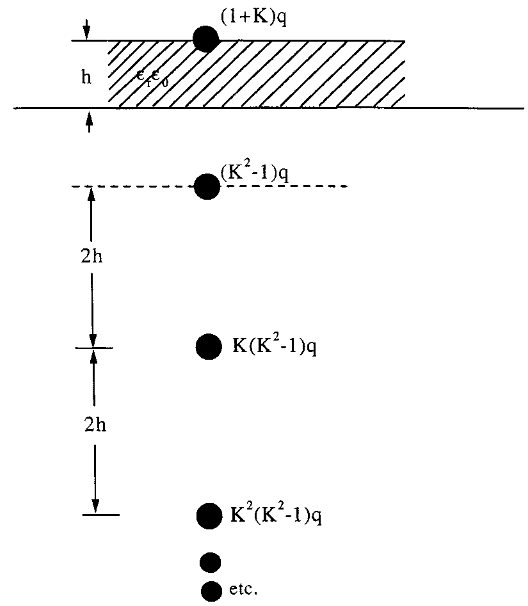

Fig. 3. Multiple images of a charge on dielectric substrate, for the field in air above the substrate.

III. Interpretation of the Formulation for the Asymptote of a Far Truncation

MoM examples have been performed for this paper using our software based on (1)–(3). Our results agree with the graphs in [1], and are "perfectly" within the tolerance of the printed width of the graphs.

Fig. 2 presents the first example. It shows that with MoM computations, *the equivalent charge density (magnitude) along the horizontal air–dielectric of the truncated case* (solid line) *is practically the same as that of the nontruncated case* (dotted line) by the image method, except for a narrow rise at the truncating point.

On the vertical truncated surface (solid line: $6.5 \leq r/h \leq 7.5$), the equivalent charge density distribution starts at the same order in magnitude as that at the nearby horizontal substrate surface, but quickly drops to a very small value before reaching the ground plane. The quick drop makes negligible the influence of the equivalent charge on the vertical surface.

The two observations above indicate that the equivalent charge on the substrate can be taken simply as that of the *regular (nontruncated) substrate* until its truncation. This means that by simply deleting the path of integration S in (1), from the truncation point w_t to infinity, we can get the reduced free charge distribution $\Delta\rho_f(\mathbf{r})$ on the strip. In other words, from (2)

$$\Delta\rho_f(\mathbf{r}) = \varepsilon_0(\varepsilon_r - 1)\int_{w_t}^{\infty} \sigma(\mathbf{r}')\frac{\partial G_p(\mathbf{r}, \mathbf{r}')}{\partial n}\, dS'. \quad (4)$$

From the data of Fig. 2, we should have $w_t > W/2 + 1.5h$, measured from the center of the strip (or equivalently $T > 1.5h$, measured from the strip edge). An integration of $\Delta\rho_f$ across the strip gives us the change of capacitance ΔC.

The change ΔC is to be subtracted from C_∞, the capacitance of a *regular* microstrip line with no truncation. The formula of the capacitance C_∞ is readily available in the Appendix. For a formula of ΔC, we need an equation for the analytical integration of (4). Fortunately, this is possible with the multiple images of the grounded substrate. This is derived in Section IV.

Equation (4) is a *far* asymptote equation, for large truncating distance of w_t. We still need a *near* asymptote equation. We have difficulty in constructing this. However, we can construct one "value" of the asymptote *right at* the edge of the strip, i.e., $w_t = W/2$ or $T = 0$. This "value" is actually a synthetic asymptote equation of the substrate thickness h and is derived in Section VI.

IV. Far Asymptote of the Potential Green's Function of a Nontruncated Substrate

We begin with the nontruncated substrate and derive the asymptote of the potential Green's function at a large distance from the charge source.

At *large field distances, the conducting strip shrinks to a line charge (i.e., point source in 2-D).* A point charge q on a grounded substrate has multiple images. Fig. 3 shows the images for the field in the free space above the substrate. Each image can be assumed to form, with a fraction of q, a dipole moment. The dipole moment of the nth image is [4], [8]

$$p_n = K^{n-1}(K^2 - 1)q \cdot 2nh \quad (5)$$

where $K = (1 - \varepsilon_r)/(1 + \varepsilon_r)$. Summing p_n in Fig. 3 and after some simple manipulations, the total dipole moment is simply reduced to

$$p = \frac{q \cdot 2h}{\varepsilon_r} \quad (6)$$

for the potential field in free space above the substrate. With the substrate grounded, there is not a net charge field, only the multipole fields. The lowest order of the latter is the dipole above with the slowest decay from the source. In other words, on the

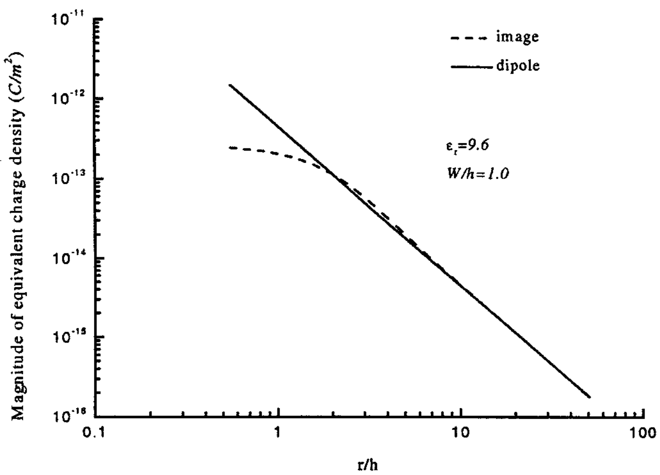

Fig. 4. Far asymptote of the equivalent charge distribution (solid line) of (8) in a log–log plot, to show its $1/r^2$ dependence. The distribution from the images (dotted line) of Fig. 2 is added for comparison.

air side of the substrate surface, the potential asymptote is the dipole field of

$$\lim_{r\to\infty}\Phi = \frac{q\cdot 2h}{2\pi\varepsilon_0\varepsilon_r}\cdot\frac{1}{r}\cdot\frac{h}{r} \tag{7}$$

where r is the distance along the substrate measured from the point charge. It may be interesting to point out that this simple formula must be formerly obtained numerically through *images*, complex [10] or classical [4]. The accuracy of this potential asymptote is discussed below.

This potential asymptote Φ is evidently proportional to the basically vertical electric field E_n in the dielectric of the non-truncated case, i.e., $E_n = \Phi/h$. Through the boundary condition of a normal electric field, we get the equivalent charge density on the dielectric surface on the air side

$$\lim_{r\to\infty}\sigma(\mathbf{r}) = -\frac{q\cdot h}{\pi\varepsilon_r}\cdot\frac{1}{r^2}\cdot\left(1-\frac{1}{\varepsilon_r}\right). \tag{8}$$

Equation (8) shows that the equivalent charge distribution decreases as $1/r^2$, where r is measured from the strip center. To emphasize this dependence, the magnitude of (8) is plotted (solid line) in the log–log scale in Fig. 4. Superimposed on it is the same equivalent charge (dashed line) from the images solution in Fig. 2 (dashed line). We see that the agreement is very good beyond the *turning point* of distance $T = (r - W/2) = 1.5h$, where T is measured from the edge of the strip.

Fig. 4 shows that, for large field distances beyond $r = W/2 + 1.5h$, a charge q and its images can simply be replaced by a vertical *point* dipole of amplitude of (6). The potential Green's function is given by (7) with an equivalent charge density given by (8).

V. Far Asymptotes of Capacitance—Substrate Truncation Far from Microstrip Line

It may be noted that equivalent charge appears differently on the opposite sides of the air–dielectric interface. Equation (8) is derived for fields on the air side. For fields on the dielectric side, the equivalent charge is the above multiplied by the factor ε_r.

We may now substitute (8) into (4) with the Green's function (1). We then get the reduction in the *free* charge density at the center of the microstrip as

$$\begin{aligned}\Delta\rho_{f,\,t} &= -2\varepsilon_0(\varepsilon_r-1)\int_{w_t}^{\infty}\sigma(\mathbf{r}')\cdot\frac{1}{2\pi\varepsilon_0 r}\cdot\frac{2h}{r}\cdot dr\\ &= \frac{2(\varepsilon_r-1)^2 qh^2}{3\pi^2\varepsilon_r^2 w_t^3}\end{aligned} \tag{9}$$

where the integration is from the truncation point w_t to infinity ∞. There is a multiplication factor of two inserted to account for the substrate truncation on *both* sides of the microstrip. Assuming the reduction of free charge is constant over the strip and of 1-V potential, we get, as mentioned before in (4), the reduction in the capacitance as

$$\Delta C_{f,\,t} = \Delta\rho_{f,\,t}\cdot W. \tag{10}$$

In other words, the *far* asymptote of capacitance of the microstrip line with distant truncated substrate is then

$$\lim_{w_t\to\infty} C_{f,\,t} = C_1 = C_\infty - \Delta C_{f,\,t} \tag{11}$$

as pointed out in Section II.

VI. Capacitance with Substrate Truncated Right at the Edge of the Microstrip Line—A Synthetic Asymptote for Different Substrate Thickness

This is the capacitance value at one truncation point and really is not an asymptote. The "value," however, is a function of the strip width, substrate thickness, and dielectric constant. It is, in fact, a synthetic asymptote of the substrate thickness.

In Fig. 1, let us assume that the substrate is truncated right on the edge of the substrate. The *near* asymptote of very thin substrate is then

$$\lim_{h\to 0} C_h(h,\,\varepsilon_r) = \frac{\varepsilon_r\varepsilon_0 W}{h}. \tag{12}$$

The second asymptote is when the substrate is very thick, meaning that the strip is quite distant from the ground plane. With the substrate still truncated at the edge of the strip, the substrate becomes a thin vertical sheet below the distant strip. There is actually mostly air between the strip and ground plane. In other words, the presence of the substrate may be neglected in this case.

The capacitance of a strip high above a ground plane at a distance h (i.e., $2h$ from source to image) in free space is given by

$$\lim_{h\to\infty} C_h(h,\,\varepsilon_r)\Big|_{\varepsilon_r=1.0} = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{8h}{W}\right)} \tag{13}$$

where the strip of width W has been made equivalent to a round wire of radius $W/4$ through conformal mapping.

A simple practice of constructing the synthetic asymptote is to simply add the two regular asymptotes together. This means that each regular asymptote not only have to behave properly in approaching its own limit, but to approaching zero or a constant at the other limit.

The first asymptote of (12) behaves properly in this fashion. The second asymptote of (13) does not. It can be made to behave properly in this fashion by taking

$$\lim_{h\to\infty} C_h(h,\,\varepsilon_r)\bigg|_{\varepsilon_r=1.0} = \frac{2\pi\varepsilon_0}{ln\left(\dfrac{8h}{W}+1\right)} - \frac{2\pi\varepsilon_0}{\dfrac{8h}{W}}. \tag{14}$$

Now the synthetic asymptote is obtained by adding (12) and (14), i.e.,

$$C_{a,\,t} = C_h|_{w_t=W/2} = \frac{2\pi\varepsilon_0}{ln\left(\dfrac{8h}{W}+1\right)} + \frac{\varepsilon_0 W}{h}\left(\varepsilon_r - \frac{\pi}{4}\right). \tag{15}$$

This is a synthetic asymptote of substrate thickness h. This synthetic asymptote is surprisingly accurate with an average error less than 1%. This accuracy is demonstrated in Figs. 5–7 in the impedances at truncations at the strip edge.

However, (15) is not a synthetic asymptote of truncation distance w_t along the substrate and measured from the center of the strip. To w_t, it is just the value of a point at $w_t = W/2$. We may call this an asymptotic point.

It should be pointed out that the free charge ρ_f in (2) and its asymptotes $\Delta\rho_f$ in (4) are integrated from the equivalent charge σ and a Green's function. Therefore, they must be proportional to voltage and, thus, a direct solution of the Laplace's equation. Since the integration of the free charge over the strip is the capacitance, *the capacitance asymptote is also a solution of the Laplace's equation*.

A solution of the Laplace's equation has to be expressible in the form of multipole expansion. As a result, we know an asymptote of the capacitance must be in a form of multipole expansion in terms of the (truncation) distance. The coefficient of the dominant term can be found by matching the capacitance at the above asymptote point. This is the asymptote at a short truncation distance.

VII. Synthetic Asymptote of Capacitance C

We shall now assemble the (regular) asymptotes of the truncation distance w_t. The first one is the C_1 with w_t approaching infinity. From Section V, the asymptote has the form

$$\lim_{w_t\to\infty} C_{f,\,t} = C_\infty - \frac{A_0}{W_t^3} \tag{16}$$

where C_∞ is taken from the Appendix, and A_0 is known from (9)–(11), i.e.,

$$A_0 = \frac{2(\varepsilon_r-1)^2 q h^2}{3\pi^2\varepsilon_r^2}\,W. \tag{17}$$

A simple choice of the synthetic asymptote may be the sum of the two regular asymptotes, far as w_t approaches infinity, and near as w_t approaches the edge of the strip. The far asymptote is (16) and contains the dc and vertical dipole term. The near asymptote toward the strip edge needs only to contain the next higher multipole. Due to the field pattern, a vertical quadrapole does not have the vertical electric field on the horizontal substrate surface to produce the equivalent charge. A vertical *octopole* does. It is, therefore, the next higher multipole with, of course, an unknown coefficient A_1. Summing the two regular asymptotes, we get the synthetic asymptote as

$$C = C_\infty - A_0/w_t^3 + A_1/w_t^5. \tag{18}$$

The coefficient A_1 of the octopole can be determined by making $C = C_{a,\,t}$ of (15) at truncation at the edge of the strip, i.e.,

$$A_1 = \left(\frac{W}{2}\right)^5 \left[C_{a,\,t} - C_\infty + \frac{A_0}{\left(\dfrac{W}{2}\right)^3}\right]. \tag{19}$$

Equation (18) is the desired synthetic asymptote of the capacitance C of a microstrip line of width W and truncating distance w_t. Its coefficients of A_0, A_1, $C_{a,\,t}$, and C_∞ are found in (17), (19), (15), and the Appendix, respectively.

The next higher order multipole beyond the vertical octopole is of the order 32, and not 16, again, because of their field patterns. We expect the pole amplitude of order 32 should be very small.

Equation (18) is for truncations at both sides of the microstrip line shown in Fig. 1. If only one side is truncated, such as a line running along the edge of a large printed circuit board, the reduction ΔC of (10) is essentially halved. This simply means that

$$C = C_\infty - \left(A_0/w_t^3 - A_1/w_t^5\right)\Big/2. \tag{20}$$

This approximation to the one-sided truncation should be more accurate than the two-sided case since only half of the multipole corrections is used in (20). This, of course, depends on the high accuracy of C_∞ of the regular (nontruncated) microstrip line shown in the Appendix. The high accuracy is found to be the case. The average error, indeed, is found to be half in the numerical results.

With (20), for all ε_r values, the characteristic impedance Z_0 and effective dielectric constant ε_{eff} of the microstrip line with a truncated substrate are easily determined by standard equations.

VIII. Results

The equivalent charge on the substrate surface in Figs. 2 and 4 has been discussed in Sections II–IV to help constructing the asymptote of far truncation in Section V. There is no need to discuss them further here.

Figs. 5–7 show and compare the results of the characteristic impedance for all truncation distances. It is observed that the characteristic impedance Z_0 and effective dielectric constant ε_{eff} of the microstrip line with a truncated substrate calculated by a synthetic asymptote technique agree with that of the MoM of Smith and Chang [1]. The error details are listed in Figs. 5–7.

The maximum errors are -6% for Z_0 and -12% for ε_{eff} at ε_r of 24, approximately at truncations about one substrate distance h from the strip edge. The average errors are taken to be less than half of the maximum values, say, -3% for the characteristic impedance. These are for the two-sided truncation case.

There is a rise in Z_0 and a lowering of ε_{eff} with closer truncations to the line. These are expected with the reduction of dielec-

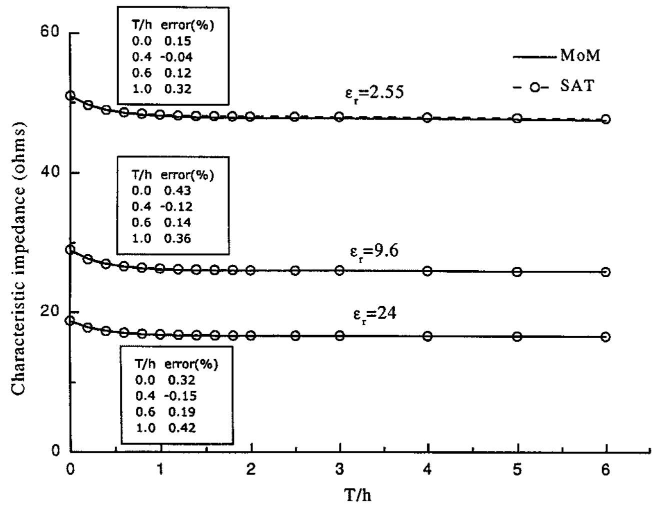

Fig. 5. Comparison of characteristic impedance of synthetic asymptote technique (SAT) and MoM for a *wide* line, $W/h = 3$, and two-sided substrate truncation. The rise in Z_0 from truncation is half as much for the one-sided case.

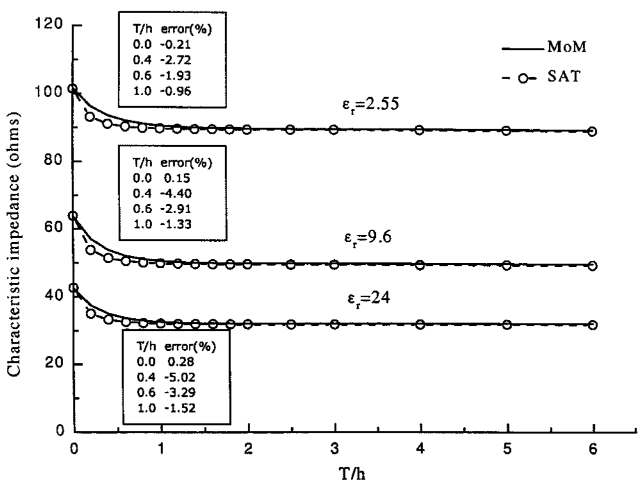

Fig. 6. Comparison of characteristic impedance of SAT and MoM for a *medium width* line, $W/h = 1$, and two-sided substrate truncation.

tric material near the line. The rises are listed with the figures. The percentage of rise in Z_0 from truncation gets larger as ε_r gets larger.

The rise in Z_0 with one-sided substrate truncation is exactly half of the two-sided case of the above. Therefore, the one-sided case is not presented in this paper's figures. As mentioned at (20), the average error is also reduced to half in the one-sided truncation, i.e., from 3% to 1.5%.

For whatever ε_r or ratio of strip width to substrate thickness W/h, the maximum errors seem to occur at a truncation distance of 0.2–0.4 T/h measured from the edge of the strip. This is the nature of synthetic asymptote—maximum error occurs somewhere between the two regular asymptotes.

The effective dielectric constant ε_{eff} or the speed of propagation is not compared with [1] in the figures. The reason is that both the formulas of Z_0 and ε_{eff} come from the same formula of C in (20) or (18). Thus, if Z_0 is found accurate, ε_{eff} must also be accurate, except with twice the error.

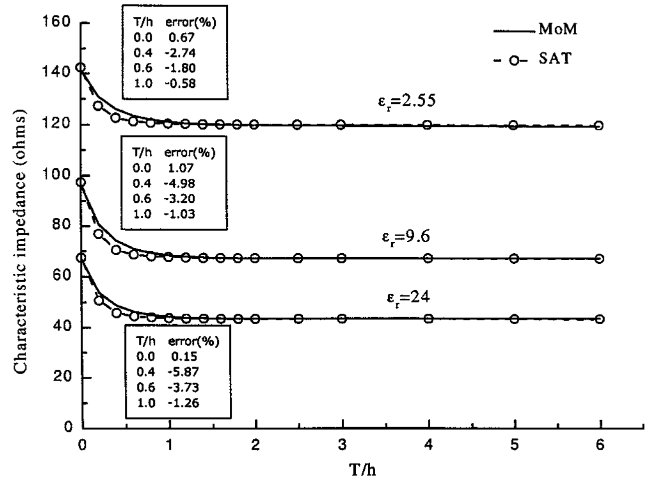

Fig. 7. Comparison of characteristic impedance of SAT and MoM for a *narrow* line, $W/h = 0.5$, and two-sided substrate truncation.

IX. Discussions

A microstrip line can be printed very close to the edge of the circuit board. As mentioned in Section I, the disturbance from the truncation at the edge of the circuit board can be easily compensated by the formulas through an optimizing routine. The *derived* formulas are simple enough that the optimizing will be very fast. This is economical both in the computation of optimizing and in the size of the printed circuit board. The accuracy will be high and the physical insight will be good. Such a possibility has been pointed out by Yamashita [3], except they had only point-fitted formulas.

As shown in Fig. 1, while the substrate is truncated, the ground plane is not. As explained in Section I-B, this arrangement may be more practical in ensuring a zero ground plane potential than that with the ground plane truncated.

Synthetic asymptote is a curve-fitting technique between two (regular) asymptotes. Bracketed by two asymptotes, the largest error must occur somewhere at the middle values of the parameter. Here, the largest error is 6% for Z_0 for the two-sided substrate truncation. For the one-sided truncation, at (20) and in the numerical results in Section VIII, the largest error is then 3%. Their averages appear to be less than half of these.

For two or more parameters of interest, a synthetic asymptote procedure has to be applied more than once. While each procedure step of synthetic asymptote or derivation may be fairly simple, there may be many steps. Confusion can be avoided if a flowchart of the steps is added, as shown in Fig. 1.

Evidently, the asymptotes have been included in arriving at many of the design formulas in the microwave literature for a long time. One example is the characteristic impedance of a microstrip line (substrate nontruncated) [11], as shown in the Appendix. Another is Kirschning and Jansen's effective dielectric constant of microstrip versus frequency [12]. The improvement is that while each of these formulas frequently has ten or more curve-fitted constants, (20) has *none*.

Strictly speaking, the first term C_∞ in (20) does have a few (say, five) such constants, but this term does not come from the synthetic asymptote of this paper, but is taken from [11]. If this term is replaced by one derived from the synthetic asymptote as that in [13], there will indeed not be any such constants. This is for an average error of 5% over the full range of substrate truncation. With just *two* curve-fitted points added [13], the average error is reduced to 2%.

The small number or void of curve-fitted constants, provides clear physical insight to the behavior of a synthetic asymptote formula. For example, in (20), the first term is the capacitance for a nontruncated substrate, and the second and third are the dipole and octopole corrections for the truncation. Similarly clear insight is found in the formulas of [13].

APPENDIX

The capacitance C_∞ of the untruncated substrate is from the microstrip formulas in the literature [11], i.e.,

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\rm eff}}} \ln\left(\frac{8h}{W} + \frac{W}{4h}\right), \qquad W/h \le 1$$

$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{\rm eff}}\left[W/h + 1.393 + 0.667\ln(W/h + 1.444)\right]}, \quad W/h \ge 1$$

$$\varepsilon_{\rm eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12h/W}}$$

giving rise to

$$C_\infty = \frac{\sqrt{\varepsilon_{\rm eff}}}{v_0 Z_0}$$

with v_0 being the light speed.

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Y. Leonard Chow (S’60–M’65) received the Ph.D. degree from the University of Toronto, Toronto, ON, Canada, in 1965.

From 1964 to 1966, he spent two years with the National Radio Astronomy Observatory, Charlottesville, VA, where he designed the array configuration of the very-large antenna (VLA) array of 27 25-m dishes of Soccoro, NM. In 1966, he was with the University of Waterloo, Waterloo, ON, Canada, where he was involved with numerical methods and simplification of electromagnetic theory for monolithic-microwave integrated-circuit (MMIC) designs. In 1996, he joined the City University of Hong Kong, Kowloon, Hong Kong, where he is currently a Professor. He has been a consultant for EEsof Inc., Westlake Village, CA, for five years. He has authored or co-authored over 250 journal and conference papers. He holds five patents in the U.S. and Canada.

Wan C. Tang received the B.S. degree from the Tsing Hua University, Beijing, China, in 1990, the M.S. degree from the Nanjing University of Science and Technology, Nanjing, China, in 1995, and is currently working toward the Ph.D. degree at the City University of Hong Kong, Kowloon, Hong Kong.

In 1999, he joined the City University of Hong Kong, where he is currently a Research Assistant. His research interests include numerical methods and simplification of electromagnetic theory for MMIC designs.